

CHAPTER

30

Measure Volume

Vocabulary:

volume
cube unit
cubic unit
layer
cubic centimetre
(cm^3)
cubic metre
(m^3)

Chapter Outcomes:

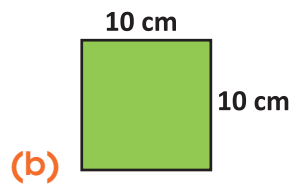
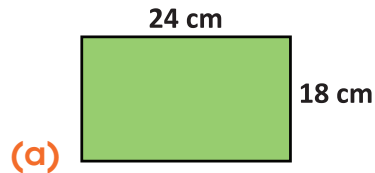
- Demonstrate an understanding of the concept of volume.
- Understand that volume can be quantified.
- Understand that capacity and volume are related.
- Understand conservation of volume.
- Develop and use proficiently the formula to calculate volume of cubes and cuboids in problem solving.
- Solve problems involving capacity and volume.



Getting Ready for Chapter 30

1. 2 litres = ____ millilitres

2. Find the area of the shapes below.



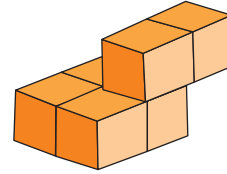
3. Name the solids below.



4. Complete the table.

Solid	Number of Faces	Number of Edges	Number of Vertices
Cube			
Cuboid			

5. Vanessa built the following shape with cubes. How many cubes are in the shape?



Describe Volume

Teaching Point 1:

How can you describe volume?

Which object takes up the most space on the shelf?



The toy box takes up the most space.

The **volume** of an object is the **measure** of the amount of space the object occupies

Activity 1:

Compare the objects. Identify the object with the greater volume.

- 1.



gift box



shoe box

2.



plastic container



lunch kit

3.



watermelon



pawpaw

4.



paint can



bucket

5.



box of toothpaste

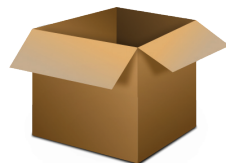


carton of milk

6.



garbage bin

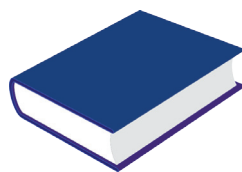


box

7.



cube



book

Explore Volume of Cubes and Cuboids

Teaching Point 1:

How can you measure volume?

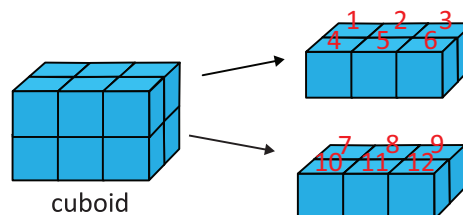
Which is the best unit to measure volume?



The cubes packs and stacks close together easily without overlaps and leaves no space.

The **cube** is the best unit to measure volume.

Find the volume of the cuboid.

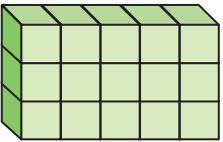
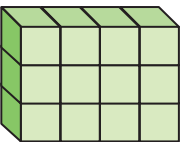
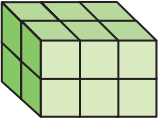
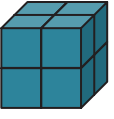
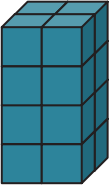
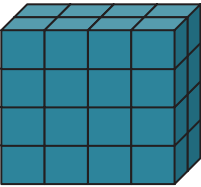
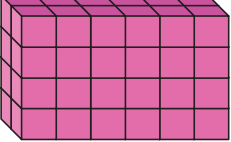
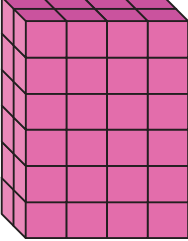
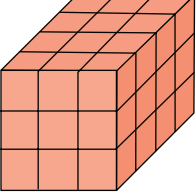


The cuboid is made up of 12 unit cubes. Its volume is 12 cubic units.

Volume is measured in **cubic units**.
Each unit cube has a volume of **1 cubic unit**.

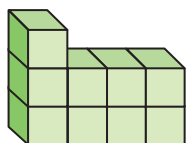
Activity 1:

Each solid is made up of unit cubes. Find the volume of each solid.

1. 	2. 
3. 	4. 
5. 	6. 
7. 	8. 
9. 	10. 

Solve.

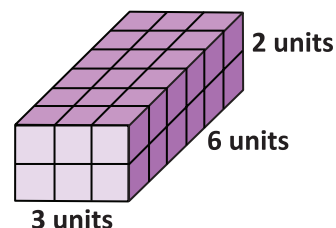
11. How many more cubes can Luke use to make the figure below a cuboid?



Teaching Point 2:

How can you use multiplication to find volume of cubes and cuboids?

Calculate the volume of the cuboid.



Length of rectangular face = 6 unit cubes

Width of rectangular face = 3 unit cubes

Height of cuboid = 2 units

Method: Use multiplication

Number of unit cubes in Layer 1:

6 unit cubes $\times$ 3 unit cubes = 18 unit cubes

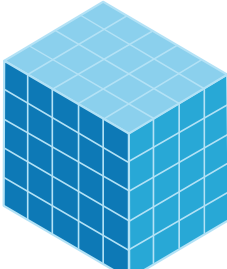
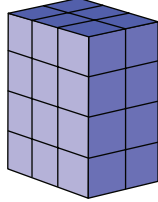
Number of unit cubes in 2 layers:

18 unit cubes $\times$ 2 unit cubes = 36 unit cubes

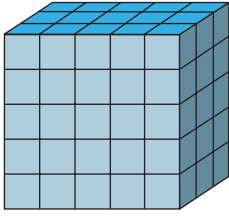
Volume of the cuboid = 36 cubic units.

Activity 2:

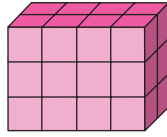
Use multiplication to find the volume of the cubes and cuboids.

1. 	2. 
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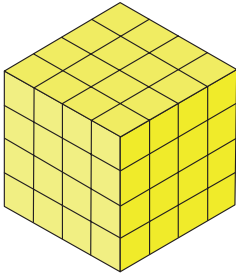
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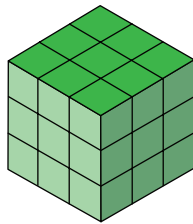
4.



5.



6.

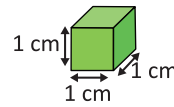


7. Jonathan covered the bottom of a box with 3 rows of 5 unit cubes. Then he stacked 4 more identical layers to fill the box.
- (a) How many unit cubes did he have in each layer?
- (b) How many unit cubes filled the box?
8. Luke covers a rectangle with 5 rows of 4 unit cubes. Then he stacks identical layers to make a cuboid. How many cubes does Lucas use to make the 4 layers?
9. Jaydn wants to make a cuboid with a length of 5 cubes, a width of 4 cubes, and a height of 3 cubes. He has 50 cubes. How many more cubes does Jaydyn need?

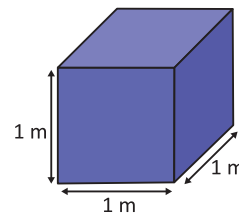
Use Standard Units to Measure Volume

Teaching Point 1:

How can you use standard units to measure volume?

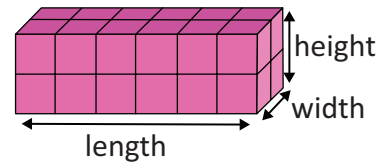


Each side of cube is 1 centimetre.
Volume of the cube = 1 cm^3



Each side of the cube is 1 metre.
Volume of the cube = 1 m^3

The solid is made up of 1 cm cubes. What is the volume of the solid?



Length = 6 cm

Width = 2 cm

Height = 2 cm

Multiply to find the area of the base:

$$6 \text{ cm} \times 2 \text{ cm} = 12 \text{ cm}^2$$

Add the number of cubes in each layer to find the volume:

$$12 \text{ cubes} + 12 \text{ cubes} = 24 \text{ cubes}$$

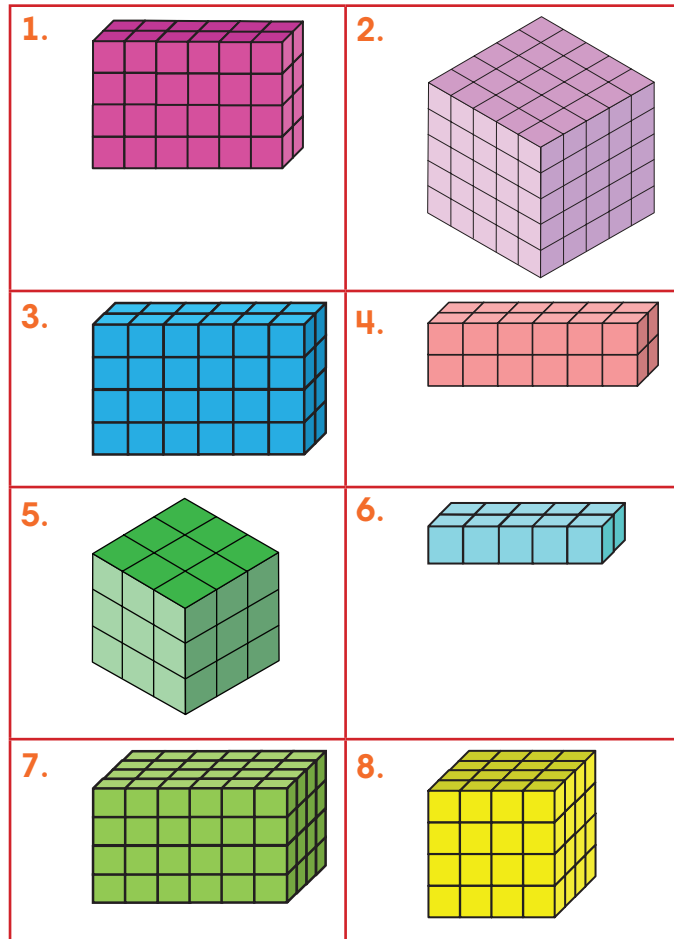
Or, multiply the base and height to find the volume.

$$\text{Volume of solid} \rightarrow 12 \text{ cm}^2 \times 2 \text{ cm} = 24 \text{ cm}^3$$

So, the volume of the solid is 24 cm^3 .

Activity 1:

Each solid is made up of 1 cm cubes. Calculate the volume of each solid.

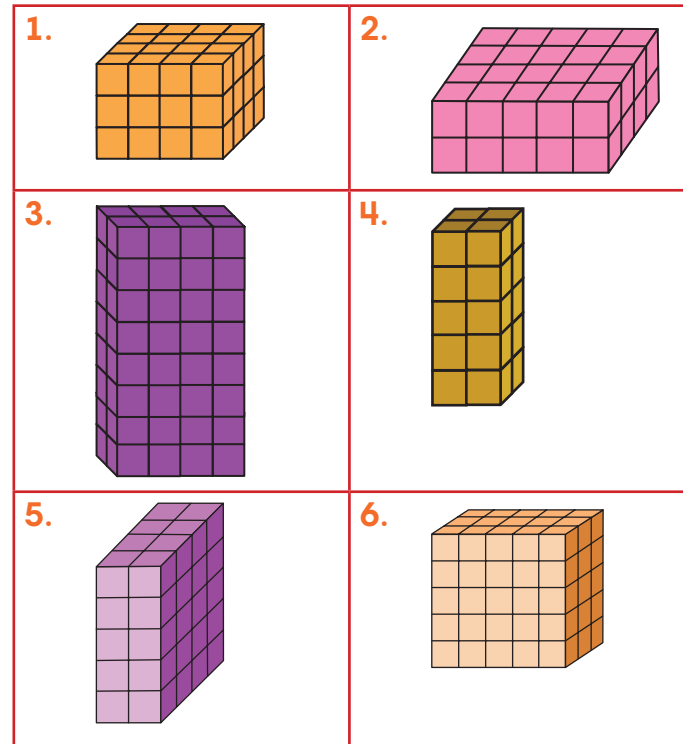


Solve.

- 1-cm cubes are stacked to fill a box. The length of the box is 12 cm, the width is 8 cm and the height is 6 cm. What is the volume of the box?
- Arun has forty 1-cm cubes. He builds a cuboid with 6 cubes in the bottom layer. He wants to have the fewest cubes left over. How many layers can Arun make?

Activity 2:

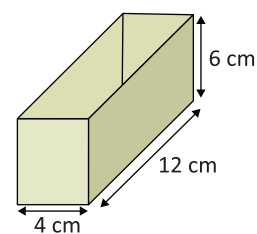
Each solid is made up of 1-metre cubes. Calculate the volume of each solid.



- How many 1 metre cubes are needed to fill this container?



- The diagram below shows an open box. What is the maximum number of 2-cm cubes that can be placed in the box.

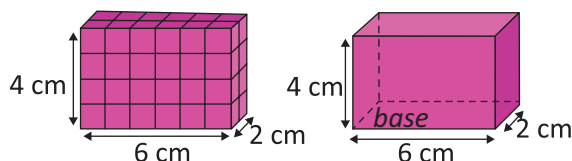


Apply Volume Formulas

Teaching Point 1:

How can you use a formula to find the volume of a cube or cuboid?

Both cuboids show the same dimensions and the same volume.



The length is 6 centimetres.

The width is 2 centimetres.

The height is 4 centimetres.

Method 1:

In **1 layer** there are $6 \times 2 = 12$ 1-cm cubes.

In **4 layers** there are $12 \times 4 = 48$ 1-cm cubes.

Volume of cuboid: $6 \times 2 \times 4 = 48 \text{ cm}^3$

Volume = length $\times$ width $\times$ height

$$V = l \times w \times h$$

Method 2:

length $\times$ width = area of base (B)

$$6 \times 2 = 12 \text{ cm}^2$$

Volume = Base $\times$ height

$$= 12 \times 4$$

$$= 48 \text{ cm}^3$$

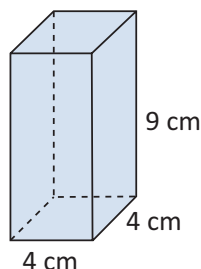
Volume = area of Base $\times$ height

$$V = B \times h$$

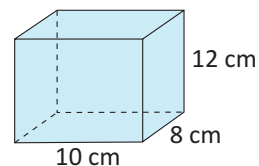
Activity 1:

Find the volume.

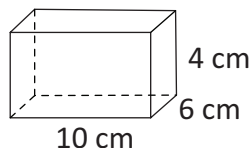
1.



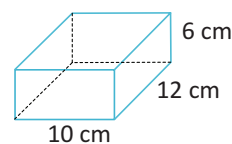
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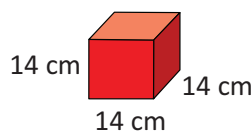
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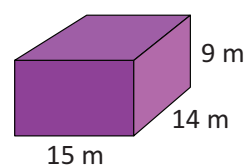
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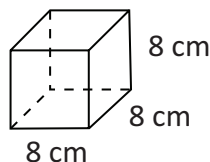
5.



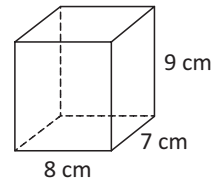
6.



7.



8.



9. A box has a length of 8 cm, a width of 4 cm and a height of 2 cm.

(a) Calculate the volume of the box.

(b) If the box is used for packaging 1-cm cubes, how many boxes are needed to pack 2 452 cubes?

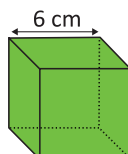
10. How many cubic centimetres of flour will a container of 30 cm by 6 cm by 4 cm hold?

11. A man is digging a hole for a swimming pool. The hole will be 9 metres long, 5 metres wide and 2 metres deep. How many cubic metres of dirt will the man have to remove?

12. A cereal box has the dimensions shown. What is the volume of the cereal box?



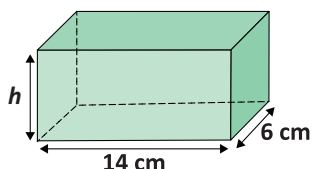
13. Find the volume of the cube below.



Teaching Point 2:

How can you use a formula to find a missing dimension?

The volume of the cuboid shown below is 336 cm^3 . The length of the cuboid is 14 cm and the width is 6 cm and the height is h cm.



Calculate the value of h .

To find the height first find the area of the base.

Base = length $\times$ width $\leftarrow$ Calculate area of base.

$$= 14 \text{ cm} \times 6 \text{ cm} \\ = 84 \text{ cm}^2$$

Volume = $B \times h$ $\leftarrow$ Write formula.

$$336 = 84 \times h \quad \leftarrow \text{Substitute.}$$

$$336 \div 84 = h \quad \leftarrow \text{Divide.}$$

$$4 = h$$

So, the length of h is 4 cm.

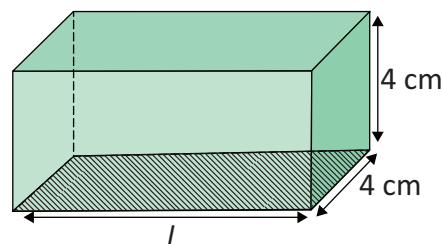
Activity 1:

Solve the problems by completing the table.

	Length	Width	Height	Volume
1.	12 cm	___ cm	4 cm	384 cm^3
2.	6 cm	5 cm	___ cm	120 cm^3
3.	___ cm	9 cm	6 cm	540 cm^3
4.	8 cm	8 cm	___ cm	384 cm^3
5.	4 cm	___ cm	11 cm	176 cm^3

6. Kayla is making a box for wooden cubes. Each wooden cube has a volume of 1 cubic centimetre. Kayla wants to have 12 cubes in each layer. If she wants to fit 60 cubes in each box, what should be the height of the box?

7. The volume of the box shown below is 96 cm^3 .



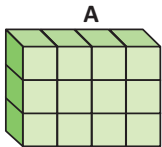
Calculate the length, l , of the box.

8. A rectangular container has a volume of 250 cm^3 . The area of its square base is 25 cm^2 . What is the height of the container?
9. Find the length of a rectangular box which has a volume of $2\,400 \text{ cm}^3$, width of 10 cm and a height of 8 cm.

Compare and Order Volume

Teaching Point 1:

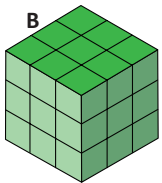
How can you compare and order volume of solids?



A

12 cubes

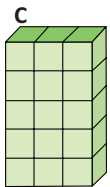
Volume = 12 cubic units



B

27 cubes

Volume = 27 cubic units



C

15 cubes

Volume = 15 cubic units

Cuboid A has the **least** volume.
Cuboid B has the **greatest** volume.

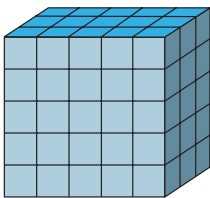
The volume of the solids in order from least to greatest is:

Solid A, Solid C, Solid B

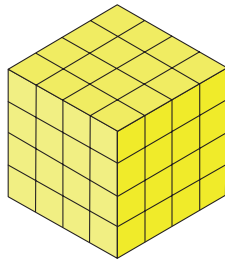
Activity 1:

Find the volume of each solid. Then compare their volumes. Use $>$, $<$ or $=$.

1.

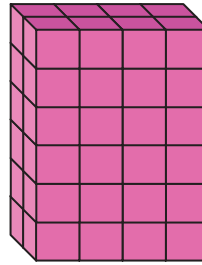


A

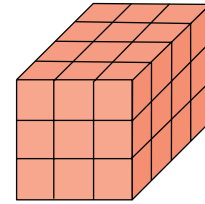


B

2.

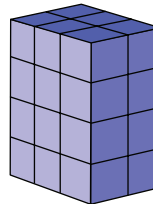


A

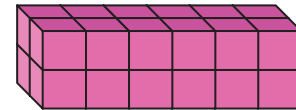


B

3.

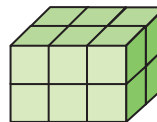


A

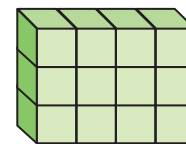


B

4.



A

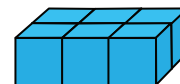


B

5.



A

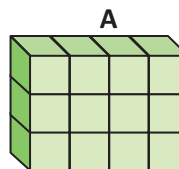


B

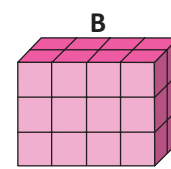
Activity 2:

Order the volume of the solids from least to greatest.

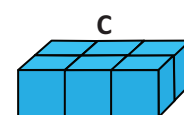
1.



A

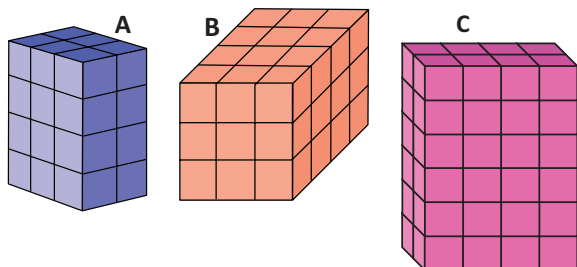


B

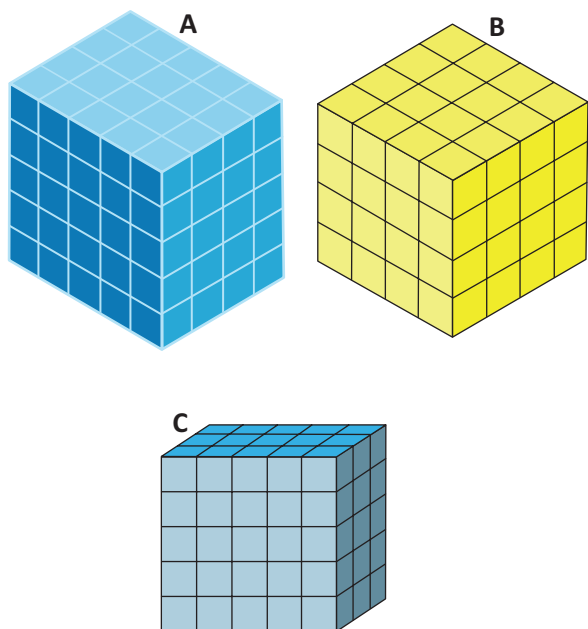


C

2.



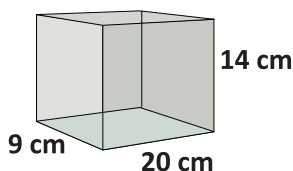
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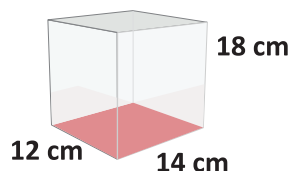
4. The volume of a box is 140 cm^3 . How does that volume compare to the volume of a cube that measures 5 centimetres on each side?

5. Two aquariums are shown below.

Aquarium 1



Aquarium 2

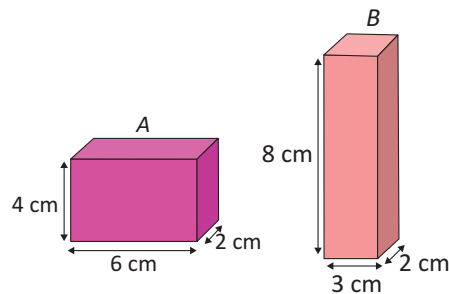


What is the difference between the volumes of the two aquariums?

Teaching Point 2:

How can you compare different solids with the same volume?

Both solids are different but they have the same volume.



Solid A

The length is 6 centimetres.

The width is 2 centimetres.

The height is 4 centimetres.

Volume of Solid A: $6 \times 2 \times 4 = 48 \text{ cm}^3$

Solid B

The length is 3 centimetres.

The width is 2 centimetres.

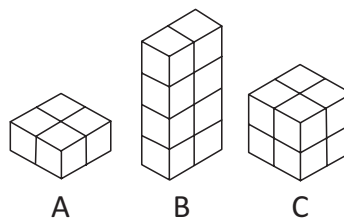
The height is 8 centimetres.

Volume of Solid A: $3 \times 2 \times 8 = 48 \text{ cm}^3$

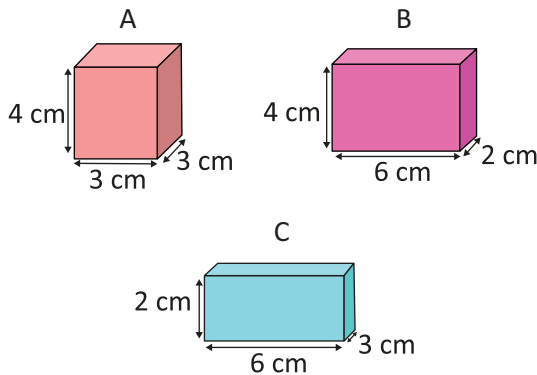
Activity 3:

Calculate the volume of the solids. Which of them has the same volume?

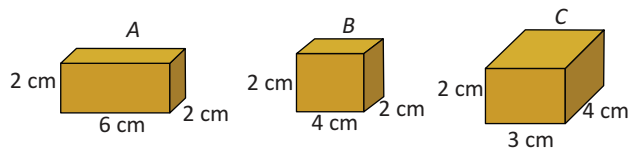
1.



2.



3. A gardener is making rectangular seed boxes. He wants the seed box to hold exactly 24 cubic centimetres of soil. Which of the following boxes can he use?



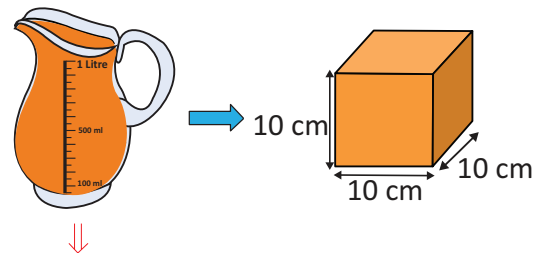
4. Aviya is building a cuboid using one-centimetre cubes. The cuboid needs to be 4 centimetres tall. How many different cuboids can Aviya make using only 16 one-centimetre cubes?
5. Matthew has 32 unit cubes. How many cuboids with different-size bases can he make if he uses all the unit cubes in each cuboid?
6. Lisa is making storage box that is 10 cm long, 12 cm wide, and 10 cm tall. She wants to build a second box that is 10 cm long and 20 cm tall. If the volume of both boxes is the same, how wide is the second box?
7. Mia makes a box that measures 10 cm by 6 cm by 4 cm. She wants to make another box with the same volume but with a different size. What can be the dimensions of the other box?

Relationship between Capacity and Volume

Teaching Point 1:

How can you relate capacity to volume?

1 litre of orange juice will completely fill a rectangular container 10 centimetres long, 10 centimetres wide and 10 centimetres in height.



1 litre = 1 000 ml

Volume of liquid in the rectangular container:

$$10 \text{ cm} \times 10 \text{ cm} \times 10 \text{ cm} = 1\,000 \text{ cm}^3$$

$$\text{So, } 1\,000 \text{ ml} = 1\,000 \text{ cm}^3$$

$$1 \text{ ml} = 1 \text{ cm}^3$$

The **capacity** of a container is the amount the container can hold.

Activity 1:

Write the following in cubic centimetres.

1. 600 ml = ___ cm^3 2. 340 ml = ___ cm^3
3. 2 L = ___ cm^3 4. 1 L 2ml = ___ cm^3
5. 14 ml = ___ cm^3 6. 455 ml = ___ cm^3

7. 1 L 300 ml = ___ cm^3

8. 3 L 34 ml = ___ cm^3

Write the following in millilitres and litres.

9. $234 \text{ cm}^3 =$ ___ 10. $1\,260 \text{ cm}^3 =$ ___

11. $7 \text{ cm}^3 =$ ___ 12. $27 \text{ cm}^3 =$ ___

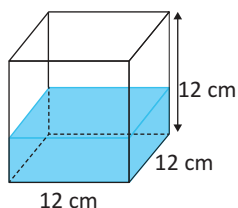
13. $3\,000 \text{ cm}^3 =$ ___ 14. $1\,004 \text{ cm}^3 =$ ___

15. $1\,490 \text{ cm}^3 =$ ___ 16. $2\,500 \text{ cm}^3 =$ ___

Teaching Point 2:

How can you relate capacity to volume?

Each edge of an aquarium is 12 cm long. The aquarium has 500 ml of water. How much more water is needed to fill the aquarium?



Capacity: $12 \times 12 \times 12 = 1\,728 \text{ cm}^3$
 $1\,728 \text{ cm}^3 = 1\,728 \text{ ml}$ or 1 L 728 ml

Volume of water in aquarium = 500 ml

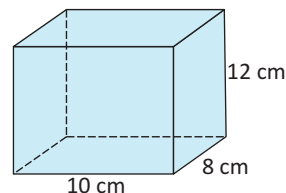
Volume of water needed:
 $1\,728 \text{ ml} - 500 \text{ ml} = 1\,228 \text{ ml}$

So, 1 L 228 ml is needed to fill the aquarium completely.

Activity 2:

Solve.

1. A rectangular container is filled with water. What is the capacity of the container? ($1\,000 \text{ ml} = 1\,000 \text{ cm}^3$)

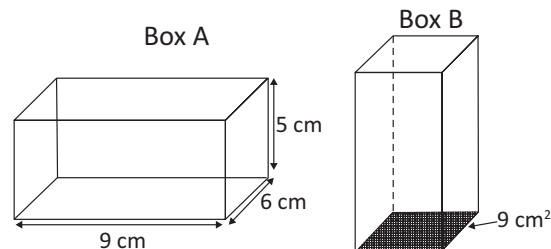


2. A rectangular container when it is $\frac{1}{2}$ full holds 4 litres of water.

(a) If 1 litre = $1\,000 \text{ cm}^3$, how many litres of water will the container hold when full?

(b) What is the height of the container if it is 40 cm long and 20 cm wide?

3. The volume of Box A is the same as the volume of Box B. What is the height of Box B if it has a square base with an area of 9 cm^2 ?



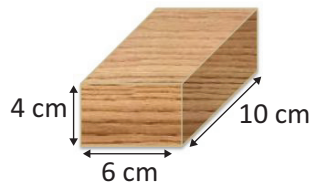
4. A container which measures 24 cm by 8 cm by 10 cm is 75% filled with water.

(a) How many litres of water are in the container?

(b) Half of the water is poured from the container to fill a second container. If the base of this second container is 12 cm by 6 cm, what is its height?

Solve Problems in Volume

1. The volume of a cube is 64 cm^3 . What is the length of each side of the cube?
2. Mickell has a rectangular block of wood measuring 10 cm by 6 cm by 4 cm.



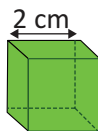
What is the most number of 2-cm cubes that can be cut from it?

3. A shipping clerk packs a cube-shaped box with side lengths of 1 metre into a container. The container's length is 12 metres, the width is 5 metres and the height is 9 metres.

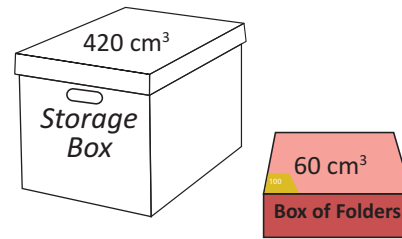


What is the volume of the space left in the container for packing?

4. 36 cubes of 2-cm sides are used to build a cuboid. The perimeter of the rectangular base of the cuboid is 32 cm.



- (a) How many cubes are at the base of the cuboid?
- (b) What is the height of the cuboid?

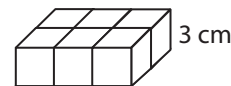


5. How many boxes of folders will fill the storage box?
6. A rectangular tank with a square base is $\frac{1}{4}$ full of water. The side length of the square base is 10 cm. When another $2\,000 \text{ cm}^3$ of water is poured into the tank, it becomes $\frac{1}{2}$ full.
 - (a) What is the capacity of the tank?
 - (b) What is the height of the tank?
7. A rectangular container is 40 centimetres long, 20 centimetres wide and 30 centimetres tall. How many litres of water is needed to fill the container?

Chapter Review

Answer the questions.

1. Jake made a cuboid as shown in the diagram using identical cubes of sides 3 cm.



What is the volume of the cuboid?

2. Leanna is using wooden cube blocks to build cuboids. If she has exactly 16 wooden cube blocks find the length, height and width to two possible cuboids she can build.

3. Arianna used unit cubes to build a solid with a length of 4 cubes, a width of 4 cubes, and height of 4 cubes. Alex built a different-sized cuboid using the same number of cubes.

(a) How many cubes did Arianna use?

(b) How many cubes did Alex use for the length, width and height of his cuboid?

4. A box is filled with toy blocks. Twenty-five blocks fit in the bottom of the box and there are 3 identical layers of blocks. What is the volume of the box?

5. Jada used unit cubes to build a cuboid with a length of 5 cubes, a width of 2 cubes, and height of 4 cubes. Jim built a cuboid with a length of 4 cubes, a width of 3 cubes and a height of 5 cubes. Who used more cubes?

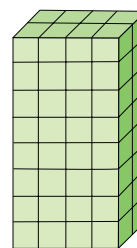
6. A baker uses a rectangular baking pan with a base measuring 36 cm^2 .



Cake batter is poured into the pan so that it is 4 cm deep. The finished cake will be 2 times as tall as the batter. What is the volume of the cake?

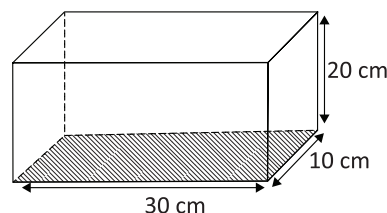
7. A rectangular block of wood that measures 16 cm by 8 cm by 4 cm is cut to form 2 cm cubic blocks. The blocks are packed into a 12 cm by 6 cm by 4 cm box. How many blocks could not be packed into the box.

8. Alisha needs 160 cubes to make a model building. The figure below shows the number of cubes in a box.



How many boxes of cubes does Alisha need to buy so she has enough cubes for her model?

9. A rectangular glass container measures 30 cm by 10 cm by 20 cm.



(a) Calculate in cm^3 the volume of the container.

(b) How many litres of water are needed to fill the container?

(c) What is the height of the water in the container when 3 000 ml of water is poured into it?

10. A water tank, as shown below, measures 60 cm by 40 cm by 30 cm. The height of the water in the tank is 20 cm. How many litres of water are needed to completely fill the tank?

